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SIMULATED ANNEALING APPROACH FOR SOLVING STOCK CUTTING PROBLEM

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ABSTRACT

In this study the simulated annealing approach for stock cutting application is studied. The conceptual approach proposed uses an energy function that measures the area of the rectangular enclosure of all the patterns to be nested, the level of similarity between pattern pairs and the amount of overlap among patterns in evaluating various pattern configurations to be generated by simulated annealing algorithm. The possibility of using three methods is addressed for pattern configuration generation. First method uses heuristics to generate initial configuration. The second and third methods employ random selection as well as random placement of patterns. Computer code is being developed to test the effectiveness of the proposed approach.

INTRODUCTION

The optimum allocation of rectangular and/or irregular patterns where a set of two-dimensional shapes is to be fitted onto a large sheet of finite dimensions frequently arises in many industries such as aerospace, shipbuilding, steel construction, shoe manufacturing, clothing, flat glass and furniture. Since 1950's, the cutting stock problem has received considerable attention as the computers have appeared as fast and economic information processors and the optimization techniques have developed. Solution of this problem is of considerable practical significance. Because of the diversity of the structures of real world stock cutting problems, there exists no general standard method for solving them. Over the years, two main approaches are emerged namely; heuristics and approaches based on linear programming relaxations.

One dimensional stock cutting problems wherein the other two dimensions of the items being cut are assumed constant are studied initially. Eisemann [1] and Haessler [2] proposed algorithms for the solution of these problems. Experience gained from the solution of one-dimensional single plate cutting stock problems is used for solving two dimensional multi-plate problems [3]. Dynamic, integer, and linear programing are used extensively as solution method. Optimization approaches proposed have restrictions in application due to NP-complete nature of the problem. To overcome this difficulty various heuristic approaches are proposed.

Albano and Sapuppo [4], Dagli [5, 6], and Dagli and Tatoglu [7] proposed heuristic algorithms which can process both rectangular and irregular patterns. The algorithm of Albano and Sapuppo [4] assumes only line segments for each side, whereas the latter two assume both line and arc segments for the sides of a pattern. Dagli and Tatoglu [7] proposed a two-stage hierarchical approach that deals with the two-dimensional allocation problems of irregular patterns in the multi-plate context. In the first stage of the procedure, initial allocation of patterns to the plates is made through mathematical programming; then based on this initial allocation detailed two-dimensional allocation is made through heuristic algorithms in the second stage.

Heuristics play an important role in the solution of stock cutting problem, since they are flexible enough to take into account various additional restrictions and objectives appearing in practice. The quality of heuristics is generally problem specific, and they can identify a pattern which is "good" for the particular problem in question. On the other hand, linear programming algorithms first solve the linear programming relaxation of the cutting pattern oriented model and search for an integer solution using complex methods or simply rounding up to the next integer.

In almost all linear programming models proposed, a two stage approach is used. Initially various cutting patterns are generated based on given stock sheet dimensions and small pattern shapes, then a decision variable is assigned for each cutting pattern and a linear programming model is formulated. Generally, linear programming approaches are not widely used in practice, and most of the time, heuristics are preferred for selecting patterns. This is basically due to the fact that the large number of possible patterns combinations (in the order of hundred millions) cannot be represented in linear programming formulations. In this study the feasibility of using simulated annealing approach to overcome solution difficulties posed by mathematical programing model is examined.

SIMULATED ANNEALING METHOD

Simulated annealing is a stochastic technique that has recently attracted significant attention as a suitable technique for optimization problems of very large scale. It has effectively solved the traveling salesman problem [10]. The method has also been successfully used for designing complex integrated circuits.

Metropolis Algorithm [8] forms the basis of the simulated annealing method. In each step of the algorithm an atom is given a small random displacement and the resulting change in the energy of the system is computed. If energy change is less than zero the displacement is accepted, and the configuration with the displaced atom is used as the starting point for the next step. If the change in energy is greater than zero then the displacement is treated probabilistically. The new configuration is accepted by comparing the Boltzmann probability and a random number between 0 and 1 generated from a uniform distribution..

The simulated annealing applied to optimization problems involves three preparatory steps. First, one must identify the analogues of the physics concepts in the optimization problem itself: the energy function becomes the objective function, the configurations of particles become the configurations of parameter values, finding a low energy configuration becomes seeking a near optimal solution, and temperature becomes the control parameter for the process. Secondly, one must select an annealing schedule consisting of a decreasing set of temperatures together with the amount of time to spend at each temperature. Third, one must have a way of generating and selecting new configurations.

Annealing schedule: Annealing schedule forms the basis of simulated annealing approach. Hence a good annealing schedule for the problem on hand is the prospect for a promising result. The annealing schedule consists of; initial temperature, temperature decrement, equilibrium condition, and stopping criteria.

Initial temperature: A large value of T is chosen. A set of initial configurations are generated. If the acceptance rate (number of accepted configuration / number of generated configuration) is less than β_0 which is a number less and close to 1, the value of T is doubled. The procedure is continued till the acceptance rate become greater than β_0 .

Equilibrium condition : Equilibrium condition is met when certain number of configuration are accepted in that temperature [12].

Temperature decrement: After each equilibrium state is reached, temperature is decremented by a constant value. Various values between 0.9 and 0.99 are used by researchers [12].

Stopping criteria: If the energy values at the end of three consecutive accepted configuration is unchanged, then it is assumed that system has reached its minimum energy level. When this condition is met the temperature is set to zero and the algorithm becomes greedy random selection algorithm which terminates at the first local minimum reached [11].

Generation of move strategy: An appropriate move generation strategy results in significant speedup in high quality solution [9]. Three different approaches is utilized in generating new configuration. In the first approach, various heuristics available are utilized to generate initial configuration. The two-dimensional allocation heuristic proposed by Dagli and Tatoglu [7] such as maximum area first and maximum arc-shaped sides first can be used in generation of the initial configuration. In the second approach, selection and placement of patterns are random. Third approach selects patterns randomly, but uses a guided nesting in placement of patterns.

Generation of initial configuration: The patterns can be nested on the finite grid in width while other dimension is infinite. The guided nesting heuristic developed can be used in placement of patterns.

Generation of new configuration: The patterns and their restricted orientation are selected randomly. Two approaches can be used. First one uses random selection and random placement strategy for all the patterns. In the second approach, a location is randomly selected on the grid containing current configuration. All the patterns having smaller coordinates than the randomly selected location remain unchanged. The remaining patterns can be nested again as described in the first approach.

SIMULATED ANNEALING APPROACH

FOR STOCK CUTTING

Energy function: The energy function for simulated annealing includes three main components to assess a given configuration. They are: area of the rectangle that encloses all the patterns, weighted sum of distances between patterns within the configurations, area of overlapping patterns.

This can be represented in functional form as:

$$E = af_1+bf_2+cf_3$$

where f_1 denotes the area of the enveloping rectangle for all patterns, and f_2 represent the amount of overlap in a given placement and finally f_3 is a weighted sum of distances of patterns within the configuration.

$$f_3 = \sum_{i=1}^{n} \sum_{j=i+1}^{n} (w_{ij}d_{ij})$$

Where d_{ij} is the distance between the patterns i and j in a given configuration and w_{ij} represents the tendency of the pattern i to attract pattern j. The values for this index for each pattern pair need to be calculated prior to the simulated annealing approach. The contribution of each component to total energy is controlled through the parameters a, b, and c that change based on the patterns allocated. Hence, it is possible to create different set of energy functions based on the irregularity of the shapes under consideration.

Pattern representation: Pattern representation is an important issue in solving stock cutting problems. Choice of efficient representation methods is important, especially for algorithms that require extensive computation time. In the proposed approach matrix representation is used. Each pattern is represented by a matrix of size K by L which is the minimum rectangular enclosure of the irregular shape. The value of K and L is determined based on the desired accuracy of computations.

Attractiveness weight generator: Simulated annealing algorithm selected requires calculation of attractiveness weights for pattern pairs. In the sample problem patterns are restricted to rotate in multiples of 90 degrees. Attractiveness weights of each pattern pair is calculated based on this restriction by evaluating all possible combinations. Rectangular enclosure is selected for each combination generated for pattern pairs. The one that gives the maximum packing density is selected as a weight to be used in the simulated annealing algorithm . The weight are given in table 1 for the patterns. The patterns used include both convex and concave arcs, and straight lines. These patterns are mapped on a grid, the corresponding grid representation of each pattern is used for allocation procedure.

1	1	2	3	4	5	6	7	8	9
2	759.0								
3	900.5	640.8							
4	726.0	878.8	558.1						
5	865.5	770.3	637.2	601.0					
6	808.3	694.3	579.9	752.0	743.6				
7	539.8	507.9	491.0	561.1	623.7	652.0			
8	604.6	613.3	600.2	537.7	642.6	638.1	456.4		
9	825.2	872.3	774.0	769.2	839.1	725.6	562.7	694.2	
10	584.2	593.8	722.7	506.1	463.4	539.2	436.8	358.3	491.6

Table 1. Attractiveness weight of Patterns

The following steps are to be taken to complete the execution of the algorithm;

1. Generate an initial configuration.
2. Calculate the initial value for the temperature.
3. Calculate new energy and the corresponding energy differences between the new and old configuration. If $\Delta E \leq 0$ go to step 6, otherwise, go to step 4.
4. Calculate the Boltzmann probability $e^{-\Delta E/T}$.
5. Pick up a random number R_n from a uniform distribution between 0 and 1. If $R_n \geq e^{-\Delta E/T}$ go to step 6, otherwise, reject the new configuration and go back to step 4.
6. Discard the old configuration and make the new configuration current.
7. Compare the energies of all configuration accepted. If the stopping criteria is met set temperature to zero and go to step 9, otherwise, go to step 8.
8. If the equilibrium condition is not met go back to step 2, otherwise, decrement temperature and go to step 2.
9. Generate a new configuration.
10. Calculate ΔE according to step 3, if $\Delta E \leq 0$ go to step 11, otherwise, reject the new configuration and go to step 9.
11. Record all the properties of the final configuration and quit.

To demonstrate the working of the algorithm under different conditions, two configurations are generated. Figure 1 corresponds to a heuristic based rule in generating the initial configuration. Figures 2 represents random selection and random movement of the patterns. Computer codes that allocates fifty patterns is underdevelopment. Parameter a, b, and c are selected based on set of patterns to be allocated. The value of these parameters will be the same for each configuration.

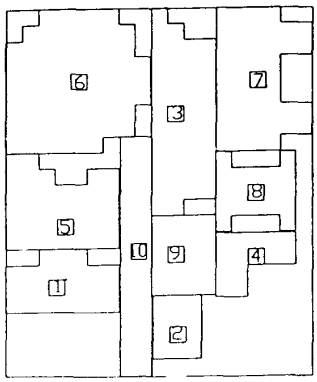

Figure 1:
Heuristic based allocation
configuration
E=439.00(a)+0(b)+264113.2(c)

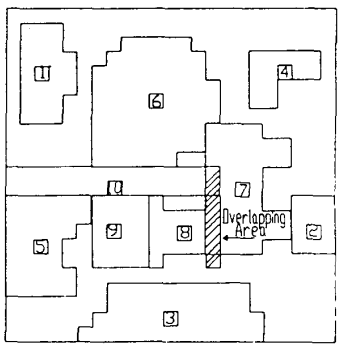

Figure 2:
Random allocation configuration
E=529.00(a)+6(b)+304214.1(c)

CONCLUDING REMARKS

The simulated annealing approach proposed in this study provided looks promising. However more experimentation and theoretical development must be sought in selection of parameters a, b, and c, based of different pattern attributes, annealing schedules and moving strategy. There are a lot of questions that remain open. After the development of computer codes it will be possible to answer some of these questions and modify the approach if required.

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